Bull. Astr. Soc. India (0000) **00**, 000–000

Physics of Supermassive Disks: Formation and Collapse

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2008 February 2

Abstract. Supermassive disks are thought to be precursors of supermassive black holes that are believed to power quasars and exist at centers of galaxies. Formation scenarios of such disks are reviewed and it is argued that gas dynamical schemes are favourable compared to stellar dynamical schemes which could however be important feeding mechanisms for the growth of the black hole. A new self-similar model of a collapse of a self-gravitating disk due to radiation induced stresses applicable to two different situations of radiative viscosity and Compton drag is presented. The collapse timescale purely due to radiative viscosity is found to be a fraction of Hubble time, $\tau_\gamma \sim \sigma_T c/(m_p G)(L_{edd}/L) \simeq 6 \times 10^9$yrs is slow and probably magnetic fields play an important role before general relativistic effects take over. A model of self-gravitating disk collapsing due to Compton drag by the Cosmic Microwave Background is also presented which is found to be effective at redshifts $1400 > z \gtrsim 300$. It is proposed that the small $\lesssim 10^5 M_\odot$ objects that form by this mechanism by $z \sim 20$ can merge and coalesce by dynamical friction to form the high redshift quasars seen. Supermassive stars which are systems (and could be end products of a supermassive disk phase) en route to the final collapse are also briefly reviewed.

Keywords : Black Holes –Formation, Supermassive Stars, Radiation Hydrodynamics, Cosmic Microwave Background.

1. Introduction

There seems to be increasing evidence that supermassive black holes are at the centers of galaxies. Dynamical searches indicate the existence of massive dark objects (MDOs) in eight systems and their masses range from $10^6 - 10^{9.5} M_\odot$ (Kormendy & Richstone 1995). Although this study does

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not confirm that the central objects are supermassive black holes, it has been inferred that the central mass is contained within 10^5 Schwarzchild radii. On an average, the black hole mass is a fraction, 10^{-2} – 10^{-3}, of the total mass of the galaxy and of order $10^{-3.5}$ of the bulge mass (Wandel 1999). Recent observations show a strong correlation between the black hole mass, $\mathfrak{m}_h$, from stellar dynamical estimates, and the velocity dispersion of the host bulges ($\mathfrak{m}_h \propto \sigma^\alpha$; where α is reported to be in the range 3.5–5; eg. Ferrarese & Merritt, 2000).

The presence of quasars at high redshifts tells us that galaxy formation had proceeded far enough for supermassive black holes to form in the standard picture (Rees 1984). A detailed model of formation of these objects should address the issues of supernovae feedback from star formation and the mechanism of efficient angular momentum transport in order to explain the massive active nuclei as early as $z = 6$. In the case of MDOs, there is a need to explain the compact sizes of $10 - 100$ pc that are implied from dynamical studies.

2. Formation scenarios

We first discuss the possibility that dense star clusters feed a seed black hole which grows initially by accreting the gas resulting from tidal disruption that occurs at the tidal radius, $R_T \simeq (6\mathfrak{m}_h/\pi\rho_*)$. Subsequently, when the Schwarzchild radius,R_s, exceeds this, the black hole then grows by swallowing low angular momentum stars whose pericenters lie within R_s. The timescale for growth during the gas release through tidal disruption suffers from serious drawbacks– the accretion rate is given by (see eg. Hills 1975), $\dot{M} = (4\pi^2 G\mathfrak{m}_h R_T/\sigma)\rho_s$, assuming a Maxwellian distribution of velocities with dispersion σ. At this rate, the growth timescale goes as $1.7 \times 10^{11}(\rho_s/10^6 M_\odot \mathrm{pc}^{-3})(\mathfrak{m}_h/M_\odot)^{1/3}(\sigma/100\mathrm{km/sec})$yrs which is large even for a dense cluster (the alternative of a seed mass of 1000 $M_\odot$ for the black hole is unlikely from theories of stellar evolution). Such models then beg the question of how clusters of densities $\sim 10^7 M_\odot/\mathrm{pc}^3$ form in the first place.

Now consider the situation where the growth proceeds by swallowing of stars. Swallowing rate of stars may be estimated in the following way. Stellar orbits diffuse by two-body relaxation toward lower angular momentum orbits until they enter a small loss-cone of semi-aperture $\theta_c \simeq (t_{dyn}/t_R)^{1/2}$ (Frank & Rees 1976, Lightman & Shapiro 1977) where t_R is the relaxation time and t_{dyn} is the dynamical time. The resulting swallowing rate is $\dot{M} \approx N_c m_*/(t_R \ln(2/\theta_c)) \approx m_*/t_{dyn}$ which is not rapid enough in most cases. The key point is that in this scenario, one must assume a extremely dense and massive cluster.

Recently, there have been proposals (Volonteri, Haardt and Madau 2003, Wyithe and Loeb 2002) motivated by quasars discovered at $z \approx 6$ that these objects have are assemblies of smaller $100 M_\odot$ objects that collapsed at $z \sim 20$ from high-σ density fluctuations. The former model invokes dynamical friction of merging systems to sink to the center as larger halo objects and involves Monte-Carlo computations based on halo merger trees from cosmological simulations in a Λ CDM cosmology. The model appears to yield the desired results of luminosity functions. This promising model for forming quasars at high red-shifts is worth exploring further to ascertain how

massive the seed black holes need to be to explain the high redshift quasars and the efficiency of mergers. Observations of ultra luminous X-ray sources (ULXs) by (Colbert & Mushotsky 1999, Makishima et al. 2000) in nearby galaxies seem to indicate that seed black holes of intermediate mass of a few hundred $M_\odot$ are possible.

There are good reasons to think that supermassive gaseous objects are remnants of a galaxy formation process. Mangalam(2001) presented a detailed physical model wherein protoquasars (or MDOs) form from a magnetized accretion of a collapsed disk, the properties of which are obtained taking into account supernovae feedback in a virialized halo. Significant star formation and supernovae activity occurs after the cloud, which is spun up by tidal torques, contracts to a radius where self-gravity is significant. The model is composed of the following stages-

1. The formation of a gaseous disk with a radial extent of about a kpc, in a host galaxy as limited by supernovae feed back. The range in halo mass for a given redshift that still retains the hot gas was calculated.

2. In previous work, gravitational instabilities in the disk was considered as the main source of viscosity. Justification was made for a magnetic viscosity and the estimated accretion rate turns out to be significant. The collapse of the disk was calculated with a generalized viscosity prescription (which includes the individual cases of magnetic, α and self-gravity induced instabilities, under a halo dominated gravitational potential into a compact central region at rapid rate of about a $M_\odot$ yr^{-1}. A self-gravitating magnetized disk solution for this central object that collapses to a seed black hole in 10^6 yrs, was calculated.

3. The implications for quasar luminosity functions and the time delay between collapse and virialization is considered in Mangalam(2003) and is based on the mass limits from cooling considerations in Mangalam(2001).

3. Collapse of supermassive disks

Here we calculate the collapse of self-gravitating compact mass that takes into account radiative stresses, which is a Newtonian 1.5 dimensional version of a quasi-spherical relativistic collapse currently under investigation. A particular application can be made to disks collapsing under angular momentum transport by radiative drag due to CMBR at high redshift. Another application is to estimate collapse time scale due to radiative viscosity after sufficient accretion of mass into a compact region of radius r_0, typically of the order of a hundred parsecs containing a mass of $10^8 M_\odot$. The problem of self-gravitating accretion flow is complicated by the coupling of Poisson's equation to the momentum and continuity equations. Clearly, its evolution has to be treated differently from the case of a prescribed background potential.

We consider a simpler model of disk where the self-consistent density distribution with a gravitational potential that is entirely due to self-gravity, is of the Mestel form $\Sigma(r,t) = v_\phi^2(t)/(2\pi G r)$, where the time dependence appears only in the rotational velocity. Taking $v_\phi = v_0\chi(t)$ and

$r = r_0\chi_1(t)\,x$ where $v_0^2 = GM_c/r_0$, and M_c is the mass out to r_0. We see that by assuming a self-similar evolution of the disk, the mass out to a given x should be independent of t and hence it follows that $\chi_1 = \chi^{-2}$ and $\Sigma = \Sigma_c\chi^4/x$ where $\Sigma_c = M_c/(2\pi r_0^2)$. From the continuity equation,

$$r\partial_t\Sigma + \partial_r(r\Sigma v_r) = 0 \tag{1}$$

we find

$$v_r = -4\chi^{-3}\dot{\chi}r_0 x. \tag{2}$$

Substituting this and the self-similar forms given above into the angular momentum equation,

$$\Sigma\partial_t v_\phi + \Sigma(v_r/r)\partial_r(rv_\phi) = (1/r^2)\partial_r(r^2\Pi_{r\phi}) \tag{3}$$

we obtain

$$\Pi_{r\phi} = -\frac{3}{2}v_0\Sigma_c r_0\chi^2\dot{\chi}, \tag{4}$$

which is *independent* of x. So far no specific viscosity mechanism has been invoked– the form of $\Pi_{r\phi}$ above is necessitated by the prescription of a Mestel disk. If a stress due radiative viscosity is assumed, $\Pi_{r\phi} = \eta_\gamma rH\frac{\mathrm{d}\Omega}{\mathrm{dr}}$, where the coefficient of radiative viscosity (Misner 1968, Weinberg 1971) $\eta_\gamma = (8/27)(\epsilon_\gamma)/(\sigma_T n_e c)$, where ϵ_γ is the photon energy density, H is the half thickness, σ_T is the Thomson cross-section and n_e is the electron density. From the energy dissipation condition, the heat flux is given by

$$\nabla\cdot\mathbf{F} \simeq \eta_\gamma\left(\frac{v_\phi}{r}\right)^2, \tag{5}$$

taking only the relevant component of the heat flux,

$$\mathbf{F} = -\frac{c}{n_e\sigma_T}\nabla p_\gamma \tag{6}$$

for Thomson scattering opacity. It follows that the half thickness, $H \propto x\chi(t)^{-3}$. Realistically the 1.5-D assumption breaks down. Nevertheless, we can push our model to get some estimates. It follows that $\eta_\gamma \propto \epsilon_\gamma/\rho c^2 \propto x$. Since it is radiation dominated, we assume a polytrope of index 4/3 and obtain

$$\frac{2}{3}\tau_\gamma\chi^2\,\dot{\chi} = \chi^{7/3}, \tag{7}$$

which leads to the solution

$$\chi(t) = \left(\frac{t}{\tau_\gamma} + 1\right)^{3/2}, \tag{8}$$

where $\chi(0) = 1$ was taken as the initial condition. The factor $\epsilon_\gamma/(\rho c^2)$ can be estimated by calculating the luminosity due to the heat flux by taking $\epsilon_\gamma = 3p_\gamma$. The collapse timescale is then given by

$$\tau_\gamma \simeq \frac{\sigma_T c}{m_p G}\frac{2}{3\pi}\frac{L_{edd}}{L}\frac{r_g}{r_0} \sim 6\times10^9\mathrm{yrs} \tag{9}$$

where $r_g = GM_c/c^2$, the gravitational radius; the fiducial value taken here corresponds to a

situation when the system is sufficiently compact and radiating at a tenth of eddington luminosity. Clearly, the model is not strictly valid when it is relativistic. The toy Newtonian model of self-similar, self-gravitating, collapsing due to radiative viscosity yields a collapse timescale, τ_γ, and suggests that the final phase of a Newtonian radiation dominated collapse to a black hole is slow. This example emphasizes the importance of other viscosity mechanisms like magnetic fields before destabilizing GR effects take over; a detailed model is currently under study.

The above model can be easily adapted to the case of collapse due to radiation drag at high redshift; the corresponding angular momentum equation can be written as

$$\Sigma\partial_t v_\phi + \Sigma(v_r/r)\partial_r(rv_\phi) = -\kappa\Sigma v_\phi, \tag{10}$$

where $\kappa(t) = \frac{4}{3}\epsilon_\gamma(t)\sigma_T/(cm_p)$ is the coefficient of Compton drag in a completely ionized plasma and $\epsilon_\gamma(t) = aT^4(t)$ is the CMBR energy density. We obtain the solution $\chi(t) = \exp\left(\int\kappa \mathrm{d}t/3\right)$, and hence $\chi_1(t) = \exp\left(-(2/3)\int\kappa \mathrm{d}t\right)$; further using $\dot z = -H_0(1+z)^{5/2}$ appropriate for a matter dominated era, the collapse factor due to Compton drag is given by

$$1/\chi_1(z) = \exp\left(\frac{16}{45}H_0^{-1}\frac{a}{m_p c}\sigma_T T_0^4\left[(1+z_i)^{5/2} - (1+z_f)^{5/2}\right]\right) = \exp\left(\frac{2\kappa(z_i)t(z_i)}{5}\left[1-\left(\frac{1+z_f}{1+z_i}\right)^{5/2}\right]\right) \tag{11}$$

which shows that the e-folding time in the angular momentum at $z_i \simeq 1400$, is initially shorter than the Hubble time by two orders of magnitude- $\kappa(z_i)t(z_i) = 2.2\times10^2\,((1+z_i)/1400)^{5/2}$, a result that is consistent with Loeb(1994) who computed a spherical model of a cloud in an expanding background (here it is assumed that the disk has turned around and collapsed). Therefore CMBR drag is effective at redshifts $1400 > z \gtrsim 300$ and it is possible to collapse smaller mass clouds $\lesssim 10^5 M_\odot$.

4. Supermassive stars

Some of the proposed scenarios are envisioned to lead to a build up of a supermassive star at relativistically compact scales as an intermediate stage of the evolution before the gravitational instability sets in and a rapid final collapse to a supermassive black hole ensues.

Supermassive stars (SMS) are equilibrium configurations that are dominated by radiation pressure (the luminosities are nearly at the Eddington limit) and can have masses between $10^4 M_\odot$ and about $10^8 M_\odot$. They are expected to be fully convective (Loeb & Rasio (1994) give a formal argument that radial entropy has to develop eventually which drives convection), isentropic and their structure can be well described by a Newtonian polytrope with $\gamma = 4/3$. The energy per nucleon in an SMS is given by the radiation entropy in units of the Boltzmann constant ($S/k \sim 0.94(M/M_\odot)^{1/2}$ where M is the mass of the star (see eg. Zeldovich & Novikov 1967). The evolution proceeds on a Kelvin-Helmholtz timescale and is driven by loss of energy and in the case of rotating SMSs, loss of angular momentum through mass-shedding. Pressure contributions from plasma components raise the adiabatic index of the equation of state,

$\Gamma = 4/3 + \beta/6$ marginally above the critical value 4/3 where β is ratio of gas to radiation pressure. General relativity leads to the existence of a maximum for the equilibrium mass (as a function of the density) and gravitation instability sets in when Γ falls below a critical value $\Gamma < \Gamma_c \approx (2/3)(2-5\eta)/(1-2\eta) + 1.12 R_s/R$ where $\eta = T/|W|$ is the ratio of the rotational to the gravitational potential energy and R is the radius of the star (Misner, Thorne & Wheeler 1973). If the plasma contribution is not enough, then the star shrinks during the evolution; rotation can however hold up the collapse if η is above a critical value. The SMS evolves on a Kelvin-Helmholtz timescale $t_{KH} = |E_c|/L_{edd} \simeq 10^9/(M/M_\odot)$yrs where $E_c = 3\times10^{54}$ergs is the equilibrium energy at the onset of gravitational instability. Typical SMSs with $M \sim 10^6 M_\odot$ have a lifetime of a 1000 yrs. Rotation can appreciably stretch their equilibrium evolution; Baumgarte and Shapiro (1999) found a lifetime independent of stellar mass of $(9\times10^{11}$s) and that key non-dimensional ratios, $R/R_s, \eta$, and $Jc/(GM^2)$ for maximally and rigidly rotating polytropes are independent of the mass, spin or radius of the star. Driven by radiation and angular momentum loss through a mass-shedding sequence, the SMSs collapse leads to a formation of a black hole through an explosion powered by hydrogen burning in CNO cycle and associated with gigantic release of neutrinos. However, this neutrino flux and resulting background from such sources is weak for even the new generation neutrino detectors like Super Kamiokande; but the possibility that about 10% of the baryons are locked in SMSs at $z < 1$ (Shi & Fuller 1998) can potentially be ruled out.

5. Summary and discussion of observational discriminants

Based on the arguments here models of monolithic formation and collapse (Mangalam 2001, 2003) of supermassive disks can explain the mass and redshift distribution of black holes of $z \lesssim 6$. However plausible merger scenarios (with dynamical friction) of smaller $\lesssim 100 M_\odot$ objects that form around $z \simeq 20$, probably through the direct collapse and supermassive star route are needed for highest redshift quasars. In the case of self-similar self-gravitating contraction due to radiation viscosity, the timescale turned out to be a fraction of Hubble time, $\tau_\gamma \sim \sigma_T c/(m_p G) = 6\times10^9$yrs; clearly, one needs to take into account GR effects which is a future goal. The case of Compton drag yielded a timescale that is two orders of magnitude smaller than the Hubble time and is effective in the range $1400 > z \gtrsim 300$. Hence, it is possible to collapse smaller mass clouds $\lesssim 10^5 M_\odot$. It worth investigating the statistics of such collapsed objects and whether mergers can produce the high redshift quasars in the picture above. The luminosity function needs to be more precisely determined to help distinguish between the models and the resulting stellar cusp profiles of the merged systems are likely to be different. The observed relation, $\mathfrak{m}_h \propto \sigma^\alpha$, with $\alpha = 4-5$ can be explained on the basis of the following physical arguments of saturation of black hole mass in (Silk & Rees 1988, Wyithe & Loeb 2002): the black hole mass saturates when luminosity impedes further accretion; ie, $L_{edd} \propto B.E./t_{dyn}$, where the gravitational binding energy scales as M^2/R resulting in $\alpha = 5$. Alternatively, though in a similar vein, the fraction of stars in an isothermal distribution that is captured by a Schwarszchild black hole is given by $f(r) \simeq (J_{\rm cap}/(2\sigma r))^2$, where $J_{\rm cap} = 4GM/c$ is the maximum angular momentum for capture; this translates into a energy flux (evaluated near the radius of influence, $r_h) \propto \rho(r_h) r_h^2 \sigma f(r_h)$ which again results in $\alpha = 5$ (Zhao, Haehnelt & Rees 2002).

In models of disk contraction that depend on self-gravity induced instabilities, the accretion is effective only upto the point when the Keplerian potential dominates over the gravity of the disk, which implies final black hole masses $\mathfrak{m}_h > 10^5 M_\odot M_{d9}$ where M_{d9} is the disk mass in units of $10^9 M_\odot$ (Mangalam 2001, Loeb & Rasio 1994). This has direct implications for the seed mass. In conclusion, our understanding of the process is only beginning and there several promising ideas that need to be further explored and more observations tests are required for discriminating amongst the models.

References

Baumgarte, A. P. & Shapiro, S. L., 1999, *Astrophys. J.*, **526**, 937.
Colbert, E. J. M., & Mushotzsky, R. F. 1999, *Astrophys. J.*, **519**, 89.
Ferrarese, L., & Merritt, D. 2000, *Astrophys. J.*, **539**, L9.
Frank, J., & Rees, M. J. 1976, *Mon. Not. R. astr. Soc.*, **176**, 633.
Gebhardt, K., Kormendy, J., Ho, L. C. et al. 2000, *Astrophys. J.*, **543**, L5
Hills, J G. *Nature*, **254**, 295.
Kormendy, J., & Richstone, D. O. 1995, *Ann. Rev. Astr. Asrtrophys.*, **33**, 581.
Lightman, A. P. & Shapiro, S. L., 1977, *Astrophys. J.*, **211**, 244.
Loeb, A., 1993 *Astrophys. J.*, **403**, 542.
Loeb, A. & Rasio 1994 *Astrophys. J.*, **432**, 52.
Makishima,K., et al 2000, *Astrophys. J.*, **535**, 632.
Mangalam, A., 2001, *Astron. Astrophys.*, **379**, 1138
Mangalam, A., 2003, in preparation.
Misner, C. W., 1968, *Astrophys. J.*, **253**, 897.
Misner, C. W., Thorne, K., & Wheeler J. A., 1968, *Gravitation*.
Rees, M. J., 1984, *Ann. Rev. Astr. Asrtrophys.*, **22**, 471
Silk, J. & Rees, M. J., 1998 *Astron. Astrophys.*, **331**, L1
Shi, X. & Fuller, G. M. 1998, *Astrophys. J.*, **503**, 307S.
Volonterri, M., Haardt, F., & Madau, P. 2003, *Astrophys. J.*, **582**, 559.
Wandel, A., 1999, *Astrophys. J.*, **519**, L39.
Weinberg, S., 1971, *Astrophys. J.*, **168**, 175.
Wyithe, J. S. B., & Loeb, A. 2002, *Astrophys. J.*, **581**, 886.
Zeldovich, Ya. B. & Novikov, I., 1967 *Relativistic Astrophysics*, vol. 1